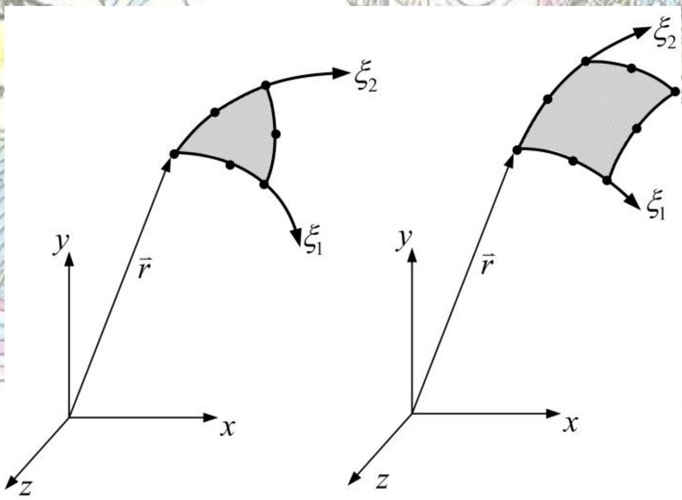
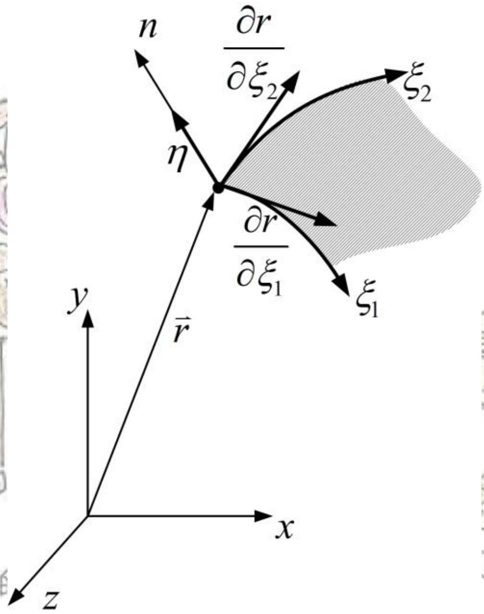
邊界元素法2維元素

解析領域為3維時，邊界元素為表面，元素為2維元素，但是領域內領域元素是3維元素。領域為2維時，邊界元素為1維，但是領域內領域元素是2維元素。通常2維元素形狀大致如下圖,可分為三角形及四角形2種。

為了解各元素內函數變化情況，須說明如何將全體座標系x，y，z上之領域表面變換成各元素局部座標系ξ_1，ξ_2，η。



2維元素



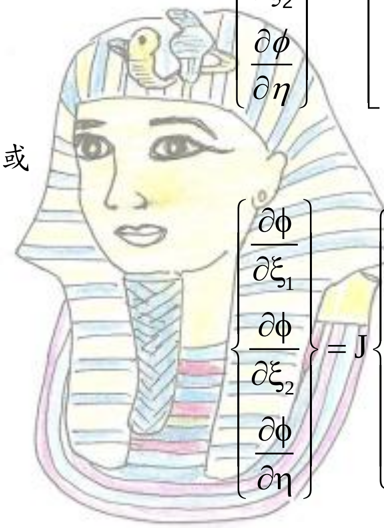
2維元素無因次座標

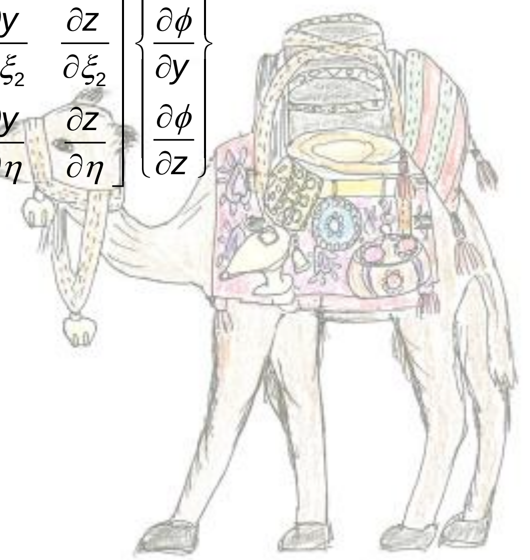
對如上圖所示座標系，函數ϕ變換法則如下式所示

$$\begin{Bmatrix} \dfrac{\partial \phi}{\partial \xi_1} \\ \dfrac{\partial \phi}{\partial \xi_2} \\ \dfrac{\partial \phi}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial x}{\partial \xi_1} & \dfrac{\partial y}{\partial \xi_1} & \dfrac{\partial z}{\partial \xi_1} \\ \dfrac{\partial x}{\partial \xi_2} & \dfrac{\partial y}{\partial \xi_2} & \dfrac{\partial z}{\partial \xi_2} \\ \dfrac{\partial x}{\partial \eta} & \dfrac{\partial y}{\partial \eta} & \dfrac{\partial z}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial \phi}{\partial x} \\ \dfrac{\partial \phi}{\partial y} \\ \dfrac{\partial \phi}{\partial z} \end{Bmatrix}$$

或

$$\begin{Bmatrix} \dfrac{\partial \phi}{\partial \xi_1} \\ \dfrac{\partial \phi}{\partial \xi_2} \\ \dfrac{\partial \phi}{\partial \eta} \end{Bmatrix} = J \begin{Bmatrix} \dfrac{\partial \phi}{\partial x} \\ \dfrac{\partial \phi}{\partial y} \\ \dfrac{\partial \phi}{\partial z} \end{Bmatrix}$$





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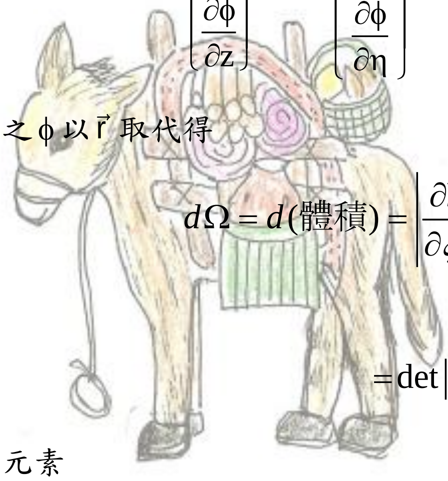
J為Jacobian矩陣。由上式可得

$$\begin{Bmatrix} \dfrac{\partial \phi}{\partial x} \\ \dfrac{\partial \phi}{\partial y} \\ \dfrac{\partial \phi}{\partial z} \end{Bmatrix} = J^{-1} \begin{Bmatrix} \dfrac{\partial \phi}{\partial \xi_1} \\ \dfrac{\partial \phi}{\partial \xi_2} \\ \dfrac{\partial \phi}{\partial \eta} \end{Bmatrix}$$

將上式之ϕ以$\vec{r}$取代得

$$d\Omega = d(\text{體積}) = \left| \frac{\partial \vec{r}}{\partial \xi_1} \times \frac{\partial \vec{r}}{\partial \xi_2} \cdot \frac{\partial \vec{r}}{\partial \eta} \right| d\xi_1 d\xi_2 d\eta$$

$$= \det |J| \, d\xi_1 d\xi_2 d\eta$$



對面積元素

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$$d(\text{體積}) = \left| \frac{\partial \vec{r}}{\partial \xi_1} \times \frac{\partial \vec{r}}{\partial \xi_2} \right| d\xi_1 d\xi_2 = |G| \, d\xi_1 d\xi_2$$

令

$$\left.\begin{aligned}\frac{\partial \vec{r}}{\partial \xi_1} &= \left(\frac{\partial x}{\partial \xi_1}, \frac{\partial y}{\partial \xi_1}, \frac{\partial z}{\partial \xi_1}\right) \\ \frac{\partial \vec{r}}{\partial \xi_2} &= \left(\frac{\partial x}{\partial \xi_2}, \frac{\partial y}{\partial \xi_2}, \frac{\partial z}{\partial \xi_2}\right)\end{aligned}\right\}$$

$|G|$表示法線向量值。

$$\vec{n} = \frac{\partial \vec{r}}{\partial \xi_1} \times \frac{\partial \vec{r}}{\partial \xi_2} = \left(\frac{\partial x}{\partial \eta}, \frac{\partial y}{\partial \eta}, \frac{\partial z}{\partial \eta}\right) = (g_1, g_2, g_3)$$

$$\left.\begin{aligned}g_1 &= \frac{\partial y}{\partial \xi_1}\frac{\partial z}{\partial \xi_2} - \frac{\partial z}{\partial \xi_1}\frac{\partial y}{\partial \xi_2} \\ g_2 &= \frac{\partial z}{\partial \xi_1}\frac{\partial x}{\partial \xi_2} - \frac{\partial x}{\partial \xi_1}\frac{\partial z}{\partial \xi_2} \\ g_3 &= \frac{\partial x}{\partial \xi_1}\frac{\partial y}{\partial \xi_2} - \frac{\partial y}{\partial \xi_1}\frac{\partial x}{\partial \xi_2}\end{aligned}\right\}$$

$$|G| = \sqrt{g_1^2 + g_2^2 + g_3^2}$$

(a) 一定元素

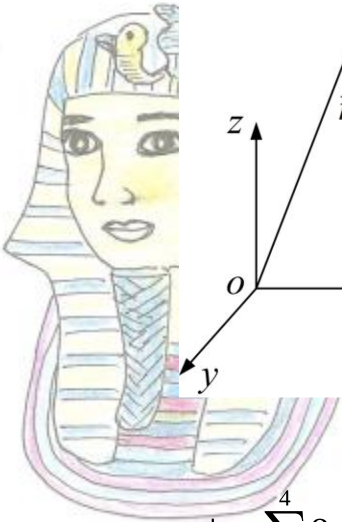
假定元素內函數值為定值，以元素重心或兩對角線交點作為節點，以其函數值作為代表值。

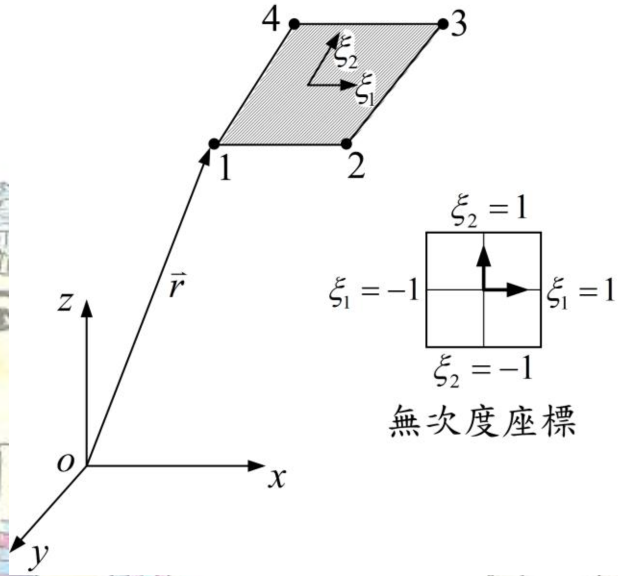
(b) 線性元素

如下圖，以四角形元素4個隅點作為節點，元素內設置從-1～1的無因次座標ξ_1及ξ_2，對函數ϕ的2次近似函數可以下式表示

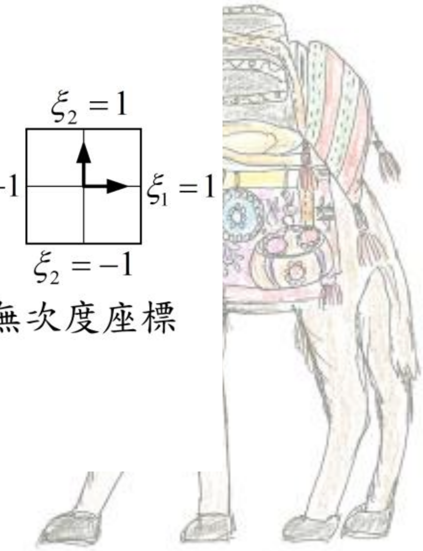
$$\phi = \alpha_1 + \alpha_2\xi_1 + \alpha_3\xi_2 + \alpha_4\xi_3$$

將各節點函數值代入上式，對α_i解之，得函數ϕ如下





四角形元素



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$$\phi = \sum_{i=1}^{4} \beta_i \phi_i$$

$$\left.\begin{aligned} \beta_1 &= \frac{1}{4}(1-\xi_1)(1-\xi_2) \quad \beta_3 = \frac{1}{4}(1+\xi_1)(1+\xi_2) \\ \beta_2 &= \frac{1}{4}(1+\xi_1)(1-\xi_2) \quad \beta_4 = \frac{1}{4}(1-\xi_1)(1+\xi_2) \end{aligned}\right\}$$

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