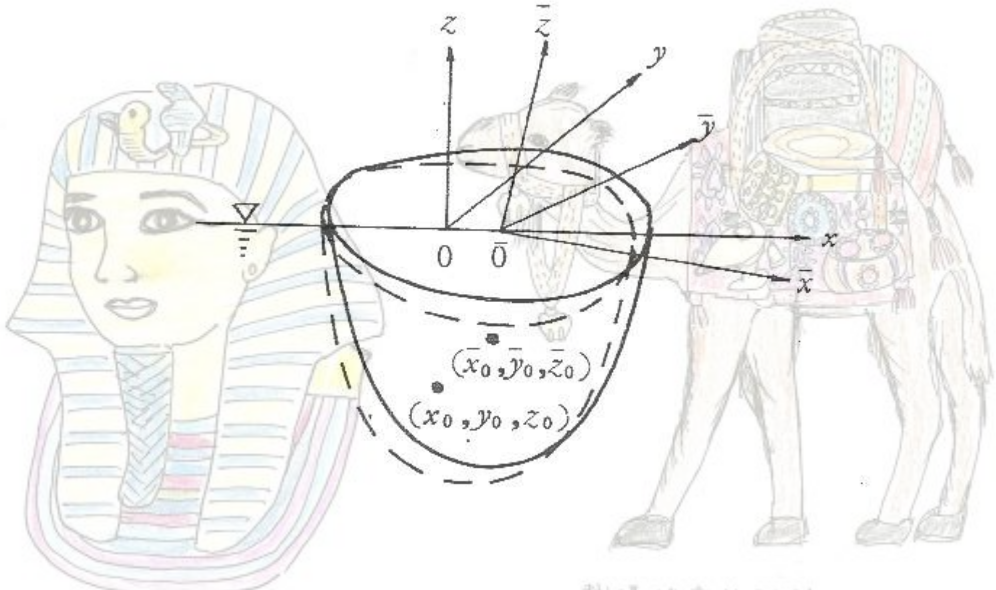
3維簡諧運動浮體運動方程式



如上圖，在水面附近有浮式結構物(floating body)存在，受波浪作用時浮體之運動可分為 x、y、z 方向直線運動水平移(surge)、橫移(sway)、垂直移(heave)及對 x、y、z 軸回轉運動橫轉(roll)、縱轉(pitch)、平轉(yaw)等 6 種運動，由於自由度有 6 度，故解析浮體運動時必須解 6 元 1 次聯立方程式。

浮體 B 浮游在水面附近，其水中部份表面積為 A，以 $\bar{x}\,\bar{y}\,\bar{z}$ 座標系固定於浮體 B，其重心位置以($\bar{x}_0, \bar{y}_0, \bar{z}_0$)表示，另外取 xyz 座標系原點固定於靜水面上，水平面內取 x, y 軸，z 軸為鉛直向上。當浮體在靜止狀態時，$\bar{x}\,\bar{y}\,\bar{z}$ 座標系重心位置與 xyz 座標系浮體重心位置(x_0, y_0, z_0)相重合。若浮體在重心附近對 x、y、z 軸分別作 $\delta_1, \delta_2, \delta_3$ 回轉，依座標變換法則可得兩座標系間第 1 近似變換如下

$$\begin{aligned}
\bar{x} &= x-(x_0-\bar{x}_0)+\delta_3(z-\bar{z}_0)-\delta_2(y-\bar{y}_0)\\
\bar{y} &= y-(y_0-\bar{y}_0)+\delta_2(x-\bar{x}_0)-\delta_1(z-\bar{z}_0)\\
\bar{z} &= z-(z_0-\bar{z}_0)+\delta_1(y-\bar{y}_0)-\delta_3(x-\bar{x}_0)
\end{aligned} \tag{1}$$

若 $z = \varsigma(x, y; t)$ 表示浮體水中部份表面方程式，則可由上式得

$$\begin{aligned}
\zeta = &-\left[(x_0-\bar{x}_0)+\delta_2(y-\bar{y}_0)-\delta_3(z-\bar{z}_0)\right]\zeta_x\\
&-\left[(y_0-\bar{y}_0)+\delta_1(z-\bar{z}_0)-\delta_2(x-\bar{x}_0)\right]\zeta_y\\
&+\left[(z_0-\bar{z}_0)+\delta_3(x-\bar{x}_0)-\delta_1(y-\bar{y}_0)\right]
\end{aligned} \tag{2}$$

$\varsigma_x, \varsigma_y, -1$ 為靜止狀態時，浮體水中部份表面 A 法線在 x、y、z 方向分量。

若流體運動具有下式所示速度勢$\Phi(x, y, z; t)$

$$\Phi(x, y, z; t) = \frac{\varsigma_0 g}{\sigma} \phi(x, y, z) e^{-i\sigma t} \tag{3}$$

ς_0 為入射波振幅，σ 為週頻率(=2π/T，T 為波週期)，
由於

$$d\zeta / dt = u\zeta_x + \upsilon\zeta_y + w\zeta_z + \zeta_t = 0 \tag{4}$$

上式可改寫為

$$\Phi_x\zeta_x + \Phi_y\zeta_y - \Phi_z + \zeta_t = 0 \tag{5}$$

從上式得浮體表面邊界條件如下：

$$\begin{aligned}
\frac{\partial \Phi}{\partial \upsilon} &= \frac{\partial x_0}{\partial t}\frac{\partial x}{\partial \upsilon} + \frac{\partial y_0}{\partial t}\frac{\partial y}{\partial \upsilon} + \frac{\partial z_0}{\partial t}\frac{\partial z}{\partial \upsilon} \\
&+ \frac{\partial \delta_1}{\partial t}\left[(z - \bar{z}_0)\frac{\partial y}{\partial \upsilon} - (y - y_0)\frac{\partial z}{\partial \upsilon}\right] \\
&+ \frac{\partial \delta_2}{\partial t}\left[(x - \bar{x}_0)\frac{\partial z}{\partial \upsilon} - (z - \bar{z}_0)\frac{\partial x}{\partial \upsilon}\right] \\
&+ \frac{\partial \delta_3}{\partial t}\left[(y - \bar{y}_0)\frac{\partial x}{\partial \upsilon} - (x - \bar{x}_0)\frac{\partial y}{\partial \upsilon}\right]
\end{aligned} \tag{6}$$

υ 為浮體表面向外法線。

流體運動為週期運動，其 x、y 及 z 軸位移及回轉振幅若分別以$\xi^*, \eta^*, \varsigma^*$ 及 $\omega_1^*, \omega_2^*, \omega_3^*$ 表示則

$$\left.\begin{aligned}
x_0 &= \bar{x}_0 + \xi^* e^{i\sigma t} \\
y_0 &= \bar{y}_0 + \eta^* e^{i\sigma t} \\
z_0 &= \bar{z}_0 + \zeta^* e^{i\sigma t}
\end{aligned}\right] \tag{7}$$

$$\left.\begin{aligned}\delta_1 &= \omega_1^* e^{i\sigma t}\\ \delta_2 &= \omega_2^* e^{i\sigma t}\\ \delta_3 &= \omega_3^* e^{i\sigma t}\end{aligned}\right] \qquad (8)$$

浮體質量為 m，通過重心與 x、y、z 軸平行慣性力距以 I_x, I_y, I_z 表示。作用於浮體壓力除重力、流體壓力 p 外，尚有復原力 R，其在 x、y、z 方向分力以 R_x, R_y, R_z 表示。復原力引起對重心力距若以 M 表示，其對 x、y 及 z 軸分力距以 M_x, M_y, M_z 表示，則浮體第 1 近似運動方程式為

$$\left.\begin{aligned}m\frac{d^2x_0}{dt^2} &= \iint_A p\frac{\partial x}{\partial \upsilon}dA + R_x\\ m\frac{d^2y_0}{dt^2} &= \iint_A p\frac{\partial y}{\partial \upsilon}dA + R_y\\ m\frac{d^2z_0}{dt^2} &= \iint_A p\frac{\partial z}{\partial \upsilon}dA + R_z\end{aligned}\right] \qquad (9)$$

$$\left.\begin{aligned}I_x\frac{d^2\delta_1}{dt^2} &= \iint_A p\left[\frac{\partial z}{\partial \upsilon}(y-\bar{y}_0) - \frac{\partial y}{\partial \upsilon}(z-\bar{z}_0)\right]dA + M_x\\ I_y\frac{d^2\delta_2}{dt^2} &= \iint_A p\left[\frac{\partial x}{\partial \upsilon}(z-\bar{z}_0) - \frac{\partial z}{\partial \upsilon}(x-\bar{x}_0)\right]dA + M_y\\ I_z\frac{d^2\delta_3}{dt^2} &= \iint_A p\left[\frac{\partial y}{\partial \upsilon}(x-\bar{x}_0) - \frac{\partial x}{\partial \upsilon}(y-\bar{y}_0)\right]dA + M_z\end{aligned}\right] \qquad (10)$$

$$\begin{aligned}R_x &= -\iint_S \rho g z_0 \frac{\partial X}{\partial \upsilon}dA\\ R_y &= -\iint_S \rho g z_0 \frac{\partial y}{\partial \upsilon}dA\\ R_z &= -\iint_S \rho g z_0 \frac{\partial Z}{\partial \upsilon}dA\end{aligned} \qquad (11)$$

$$\left.\begin{aligned}
M_x &= -\iint_A \rho g \delta_1 (y-\bar{y}_0)\left[\frac{\partial z}{\partial \upsilon}(y-\bar{y}_0)-\frac{\partial y}{\partial \upsilon}(z-\bar{z}_0)\right]dA \\
M_y &= -\iint_A \rho g \delta_2 (x-\bar{x}_0)\left[\frac{\partial x}{\partial \upsilon}(z-\bar{z}_0)-\frac{\partial z}{\partial \upsilon}(x-\bar{x}_0)\right]dA \\
M_z &= -\iint_A \rho g \delta_3 (z-\bar{z}_0)\left[\frac{\partial y}{\partial \upsilon}(x-\bar{x}_0)-\frac{\partial x}{\partial \upsilon}(y-\bar{y}_0)\right]dA
\end{aligned}\right] \tag{12}$$

將(7)、(8)式代入(9)、(10)式，又因

$$\frac{p}{\rho g \varsigma_0} = i\phi \exp(-i\sigma t) \tag{13}$$

演算後得

$$\left.\begin{aligned}
\frac{\xi^*}{\varsigma_0} &= -a_1 \iint_A i\phi \frac{\partial x}{\partial \upsilon} dA \\
\frac{\eta^*}{\varsigma_0} &= -a_2 \iint_A i\phi \frac{\partial y}{\partial \upsilon} dA \\
\frac{\varsigma^*}{\varsigma_0} &= -a_3 \iint_A i\phi \frac{\partial z}{\partial \upsilon} dA
\end{aligned}\right] \tag{14}$$

$$\left.\begin{aligned}
\frac{\omega_1^*}{\varsigma_0} &= -\alpha_1 \iint_A i\phi \left[\frac{\partial z}{\partial \upsilon}(y-\bar{y}_0)-\frac{\partial y}{\partial \upsilon}(z-\bar{z}_0)\right]dA \\
\frac{\omega_2^*}{\varsigma_0} &= -\alpha_2 \iint_A i\phi \left[\frac{\partial x}{\partial \upsilon}(z-\bar{z}_0)-\frac{\partial z}{\partial \upsilon}(x-\bar{x}_0)\right]dA \\
\frac{\omega_3^*}{\varsigma_0} &= -\alpha_3 \iint_A i\phi \left[\frac{\partial x}{\partial \upsilon}(x-\bar{x}_0)-\frac{\partial z}{\partial \upsilon}(y-\bar{y}_0)\right]dA
\end{aligned}\right] \tag{15}$$

$$\left.\begin{aligned}
a_1 &= a_2 = \frac{\rho}{m\sigma^2 / g} \\
a_3 &= \frac{\rho}{m\sigma^2 / g + R_z / g}
\end{aligned}\right]$$

$$
\left.
\begin{aligned}
\alpha_1 &= \frac{\rho}{I_x\sigma^2 / g + M_x / \delta_1} \\
\alpha_2 &= \frac{\rho}{I_y\sigma^2 / g + M_y / \delta_2} \\
\alpha_3 &= \frac{\rho}{I_z\sigma^2 / g}
\end{aligned}
\right]
$$

將(7)、(14)、(15)代入(6)得

$$
\begin{aligned}
\frac{\partial\phi}{\partial\upsilon} = -\frac{\sigma^2}{g}\Bigg\{ & a_1 \frac{\partial x}{\partial\upsilon}\iint_A \phi \frac{\partial x}{\partial\upsilon} dA + a_2 \frac{\partial y}{\partial\upsilon}\iint_A \phi \frac{\partial y}{\partial\upsilon} dA + a_3 \frac{\partial z}{\partial\upsilon}\iint_A \phi \frac{\partial z}{\partial\upsilon} dA \\
& + \alpha_1 \left[\frac{\partial z}{\partial\upsilon}\left(y - \bar{y}_0\right) - \frac{\partial y}{\partial\upsilon}\left(z - \bar{z}_0\right)\right]\iint_A \phi \left[\frac{\partial z}{\partial\upsilon}\left(y - \bar{y}_0\right) - \frac{\partial y}{\partial\upsilon}\left(z - \bar{z}_0\right)\right] dA \\
& + \alpha_2 \left[\frac{\partial x}{\partial\upsilon}\left(z - \bar{z}_0\right) - \frac{\partial z}{\partial\upsilon}\left(x - \bar{x}_0\right)\right]\iint_A \phi \left[\frac{\partial x}{\partial\upsilon}\left(z - \bar{z}_0\right) - \frac{\partial z}{\partial\upsilon}\left(x - \bar{x}_0\right)\right] dA \\
& + \alpha_3 \left[\frac{\partial y}{\partial\upsilon}\left(x - \bar{x}_0\right) - \frac{\partial x}{\partial\upsilon}\left(y - \bar{y}_0\right)\right]\iint_A \phi \left[\frac{\partial y}{\partial\upsilon}\left(x - \bar{x}_0\right) - \frac{\partial x}{\partial\upsilon}\left(y - \bar{y}_0\right)\right] dA \Bigg\}
\end{aligned}
$$

若將浮體水面下表面以 Ns 個一定面元素分割，可將上式以下列矩陣形式表示，即得浮體表面上勢函數與導函數間的 1 次關係式。

$$
\{\bar{\phi}\} = [T]\{\phi\} \quad , \bar{\phi} = \partial\phi / \partial\upsilon
$$

$$
t^*_{ij} = a^*_{ij} + b^*_{ij} + b^*_{ij} + d^*_{ij} + e^*_{ij} \qquad \text{(I, j=1,2, } \cdots \text{，Ns)}
$$

$$
a^*_{ij} = -a_1 \frac{\sigma^2}{g}\left[\left(\frac{\partial x}{\partial\upsilon}\right)_j \left(\frac{\partial x}{\partial\upsilon}\right)_i + \left(\frac{\partial y}{\partial\upsilon}\right)_j \left(\frac{\partial y}{\partial\upsilon}\right)_i\right]\Delta A_j
$$

$$
b^*_{ij} = -a_3 \frac{\sigma^2}{g}\left[\left(\frac{\partial z}{\partial\upsilon}\right)_j \left(\frac{\partial z}{\partial\upsilon}\right)_i\right]\Delta A_j
$$

$$c_{ij}^{*} = -\alpha_1 \frac{\sigma^2}{g}\left[\frac{\partial z}{\partial \upsilon}\left(y-\bar{y}_0\right)-\frac{\partial y}{\partial \upsilon}\left(z-\bar{z}_0\right)\right]_j\left[\frac{\partial z}{\partial \upsilon}\left(y-\bar{y}_0\right)-\frac{\partial y}{\partial \upsilon}\left(z-\bar{z}_0\right)\right]_i \Delta A_j$$

$$d_{ij}^{*} = -\alpha_2 \frac{\sigma^2}{g}\left[\frac{\partial x}{\partial \upsilon}\left(z-\bar{z}_0\right)-\frac{\partial z}{\partial \upsilon}\left(x-\bar{x}_0\right)\right]_j\left[\frac{\partial x}{\partial \upsilon}\left(z-\bar{z}_0\right)-\frac{\partial z}{\partial \upsilon}\left(x-\bar{x}_0\right)\right]_i \Delta A_j$$

$$e_{ij}^{*} = -\alpha_3 \frac{\sigma^2}{g}\left[\frac{\partial y}{\partial \upsilon}\left(x-\bar{x}_0\right)-\frac{\partial x}{\partial \upsilon}\left(y-\bar{y}_0\right)\right]_j\left[\frac{\partial y}{\partial \upsilon}\left(x-\bar{x}_0\right)-\frac{\partial x}{\partial \upsilon}\left(y-\bar{y}_0\right)\right]_i \Delta A_j$$

ΔA_j 為各表面離散元素面積。

作用於浮體強制力及其力矩如下

$$F_x = \rho g \varsigma_0 \iint_{A_5} i\phi \frac{\partial x}{\partial \upsilon} dA e^{-i\sigma t}$$

$$F_y = \rho g \varsigma_0 \iint_{A_5} i\phi \frac{\partial y}{\partial \upsilon} dA e^{-i\sigma t}$$

$$F_z = \rho g \varsigma_0 \iint_{A_5} i\phi \frac{\partial z}{\partial \upsilon} dA e^{-i\sigma t}$$

$$M_x = \rho g \varsigma_0 \iint_{A_5} i\phi \left[\frac{\partial z}{\partial \upsilon}(y-\bar{y}_0)-\frac{\partial y}{\partial \upsilon}(z-\bar{z}_0)\right] dA e^{-i\sigma t}$$

$$M_y = \rho g \varsigma_0 \iint_{A_5} i\phi \left[\frac{\partial x}{\partial \upsilon}(z-\bar{z}_0)-\frac{\partial z}{\partial \upsilon}(x-\bar{x}_0)\right] dA e^{-i\sigma t}$$

$$M_z = \rho g \varsigma_0 \iint_{A_5} i\phi \left[\frac{\partial y}{\partial \upsilon}(x-\bar{x}_0)-\frac{\partial x}{\partial \upsilon}(y-\bar{y}_0)\right] dA e^{-i\sigma t}$$